Orientation & Rotation 0x01

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Our Journey

1. Theory

Orientation and Rotation

- DoF
- Definition
- Rotation
- Representations

2D

- Polar Coordinate(angle)
- Complex Number
  - Euler Formula
  - Taylor expansion
  - rotation composition

Lie Theory

- Lie Group
- Lie algebra
- SO(3)
- SE(3)

3D

- Euler angle
  - Gimble lock
- Rotation Vector & Axis-angle
- Rodrigues Formula
- **Euler parameters**
- **Quaternions**

2. Coding(If there is a enough time...)

- Visual Studio(XCode)
- Opengl
- C/C++

Why Do We learn the quaternion

장점

1. 알고 있으면 있어 보인다
2. Gimble lock 없다
3. Rotation을 interpolation 할 수 있다(Slerp)
4. 연산이 효율적이다
5. 메모리 아낄 수 있다(9 -> 4)
6. 컴퓨터 그래픽스, 로보틱스, 게임, 어플 개발 등 많이 쓴다

단점

1. **이해하기 쉽지 않다**

Orientation && Rotation

Orientation & Rotation: DoF

Questions

What is the DoF of 2D rotation?

What is the DoF of 3D rotation?

What is the DoF of 4D rotation?

What is the DoF of nD rotation?

Orientation & Rotation: DoF

Questions

What is the DoF of 2D rotation?
ans) 1

What is the DoF of 3D rotation?
ans) 3

What is the DoF of 4D rotation?
ans) 6

What is the DoF of nD rotation?
ans) nC2

Orientation & Rotation: Definition

What is the Rotation?

Rotation

Article Talk Read Edit View h

From Wikipedia, the free encyclopedia

This article is about movement of a physical body. For other uses, see Rotation (disambiguation).

"Rotate" redirects here. For the song, see Rotate (song). For the ghost town, see Rotate, Kansas.

This article **needs additional citations for verification**. Please help improve this article by adding citations to reliable sources. Unsourced material may be challenged and removed.

Find sources: "Rotation" – news · newspapers · books · scholar · JSTOR *(March 2014) (Learn how and when to remove this message)*

Rotation or **rotational motion** is the circular movement of an object around a central line, known as an ***axis of rotation***.

https://en.wikipedia.org/wiki/Rotation

Orientation & Rotation: Definition

What is the Orientation?

Orientation (geometry)

文A 16 languages

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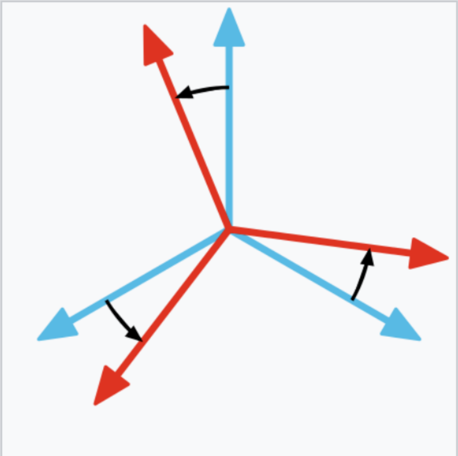
From Wikipedia, the free encyclopedia

This article is about the orientation or attitude of an object or a shape in a space. For the orientation of a space, see Orientability.

In geometry, the **orientation**, **attitude**, **bearing**, **direction**, or **angular position** of an object – such as a line, plane or rigid body – is part of the description of how it is placed in the space it occupies.[1] More specifically, it refers to the imaginary rotation that is needed to move the object from a reference placement to its current placement. A rotation may not be enough to reach the current placement, in which case it may be necessary to add an imaginary translation to change the object's position (or linear position). The position and orientation together fully describe how the object is placed in space. The above-mentioned imaginary rotation and translation may be thought to occur in any order, as the orientation of an object does not change when it translates, and its position does not change when it rotates.



Changing orientation of a rigid body is the same as rotating the axes of a reference frame attached to it.

Euler's rotation theorem shows that in three dimensions any orientation can be reached with a single rotation around a fixed axis. This gives one common way of representing the orientation using an axis-angle representation. Other widely used methods include rotation quaternions, rotors, Euler angles, or

https://en.wikipedia.org/wiki/Orientation_(geometry)

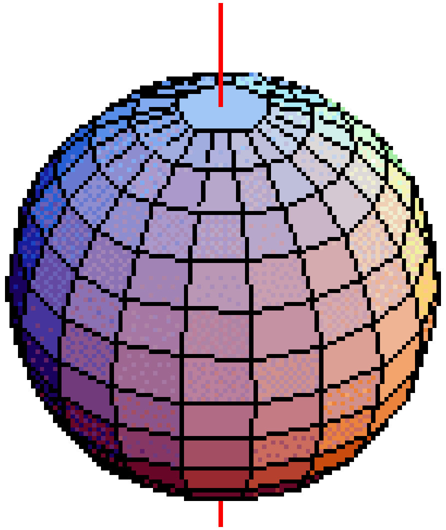
Orientation & Rotation: Definition

Rotation(Vector): Circular movement

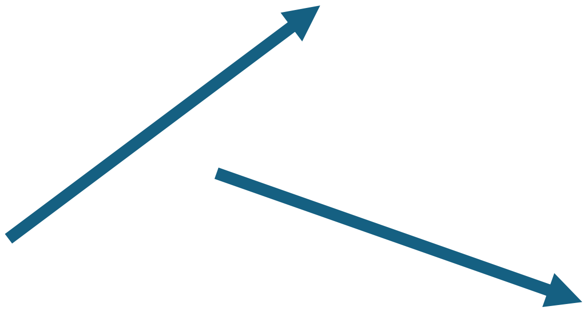
- It describes the action of turning an object around an axis.

Orientation(Position): State

- It describes the state of an object's direction after it has been rotated.



<Rotation>



<Orientation>

https://en.wikipedia.org/wiki/Rotation

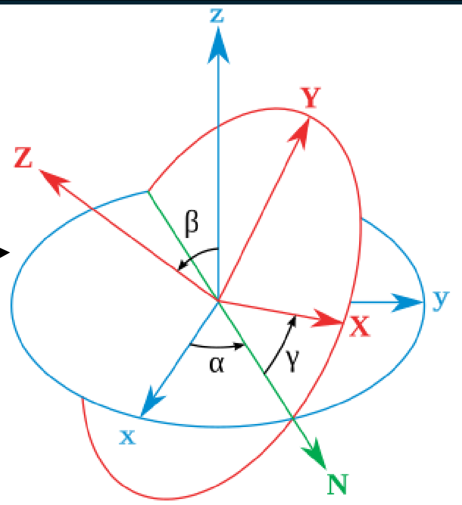
Orientation & Rotation

There are many ways to describe the rotation

1. Rotation Matrix
2. Euler angle
3. Axis-angle
4. Rodrigues Formula
5. Rotation Vector
6. Unit Quaternion
   (Euler Parameters)

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

<1>



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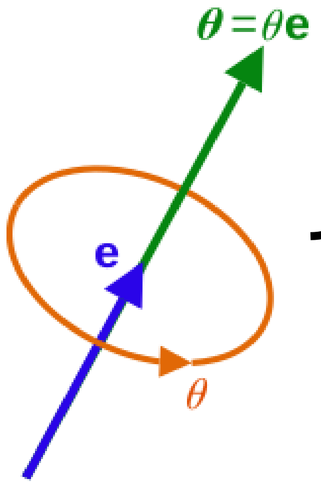


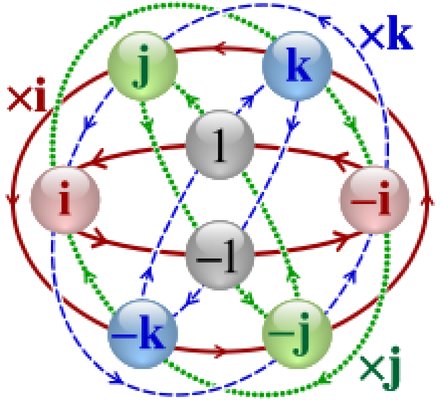
<3>

$$\mathbf{v}_{\mathrm{rot}} = \mathbf{v} + (\sin\theta)(\mathbf{e}\times\mathbf{v}) + (1-\cos\theta)(\mathbf{e}\times(\mathbf{e}\times\mathbf{v}))$$

<4>

$$\begin{bmatrix} 0 \\ 0 \\ \frac{\pi}{2} \end{bmatrix}$$

<5>

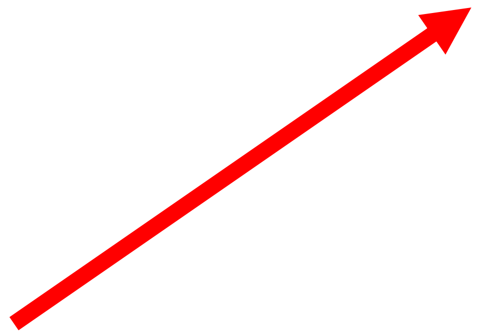


<6>

https://en.wikipedia.org/wiki/

2D

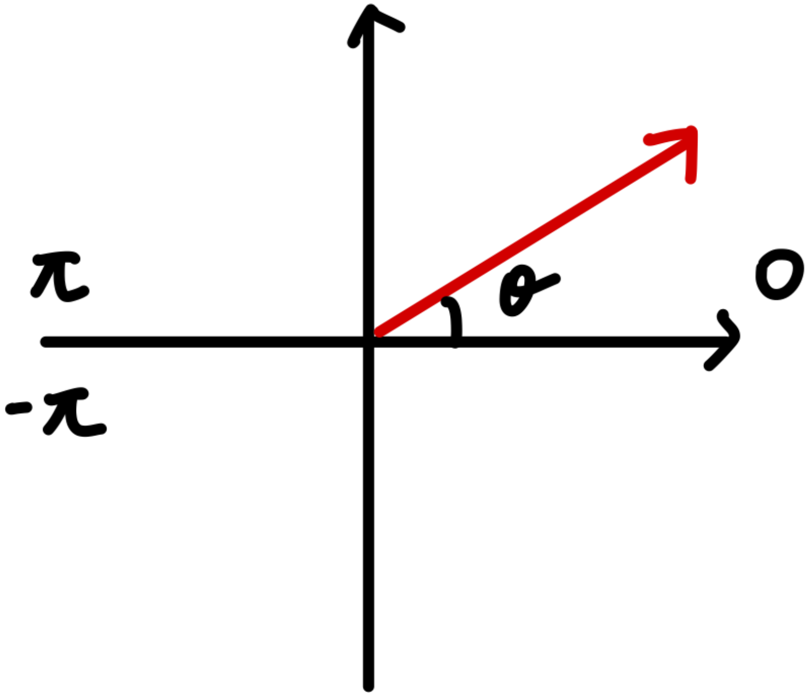
Orientation: 2D



How to express this orientation?

Orientation: 2D

Polar Coordinate(angle)

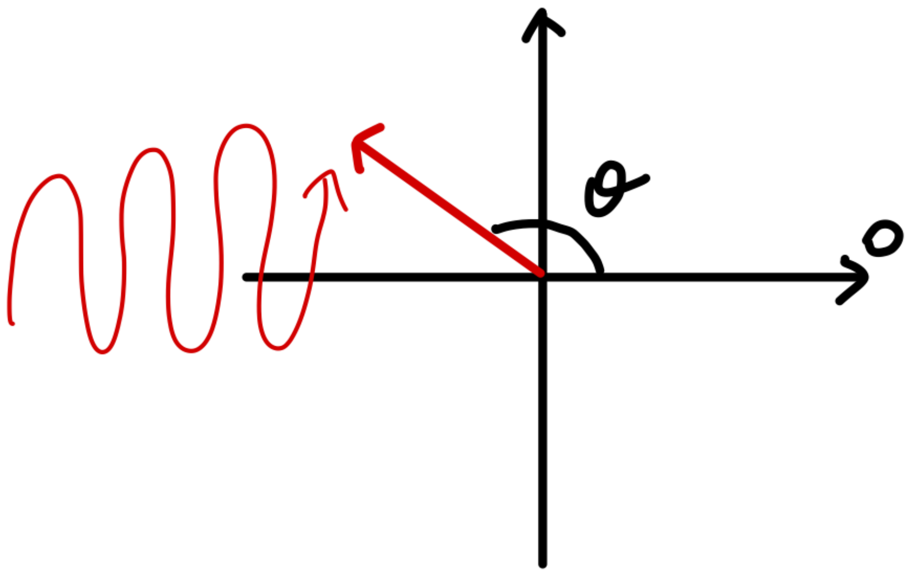


Orientation: θ

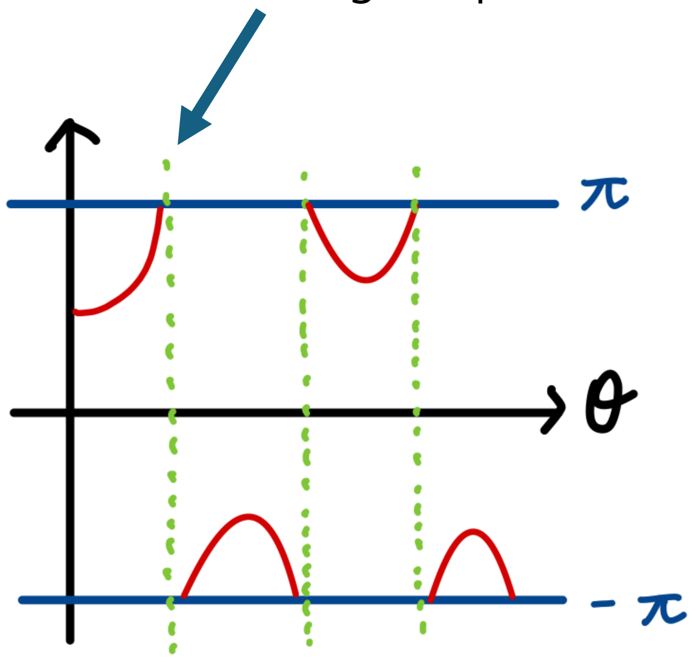
What is the problem?

Orientation: 2D

Polar Coordinate(angle)



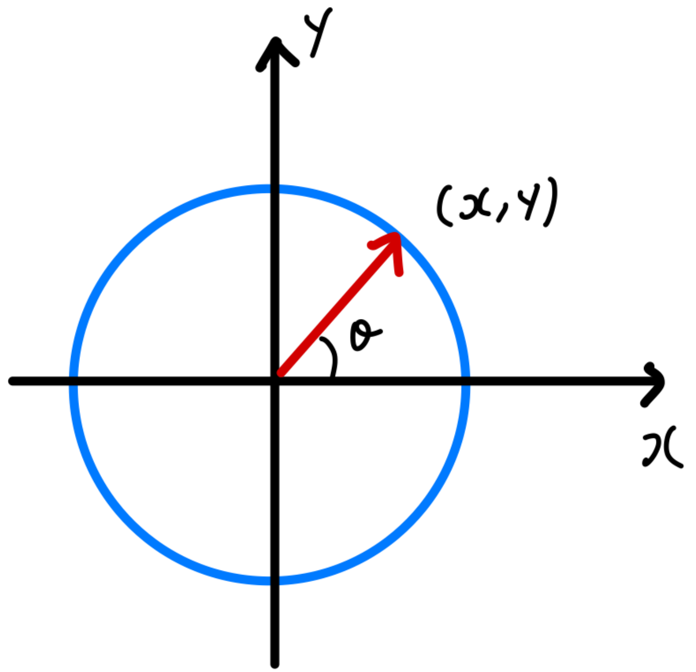
<Continuous Motion>



but… Representation is not continuous

Orientation: 2D

Cartesian Coordinate



<Cartesian Coordinate>

What is the problem?

: Over-parameterized

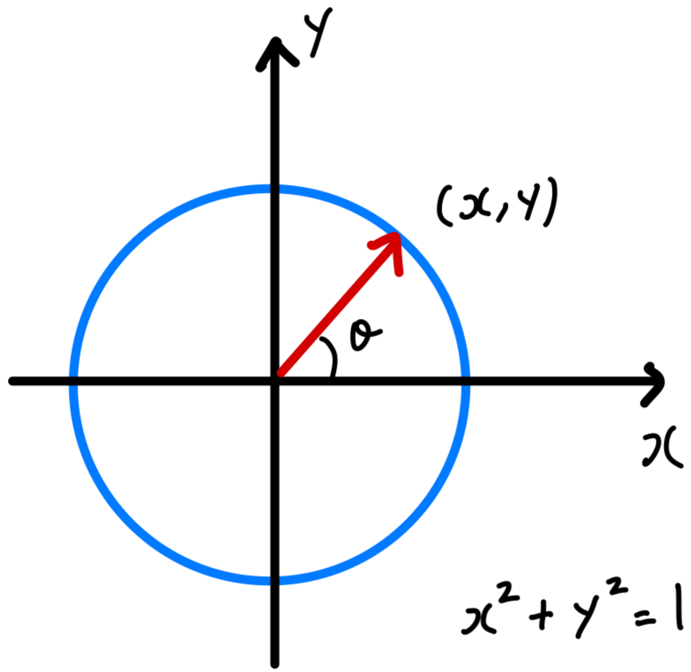
DoF: 1 -> 2

How can we Solve this problem?

: Constraint

Orientation: 2D

Cartesian Coordinate: Rotation matrix



<Cartesian Coordinate>

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

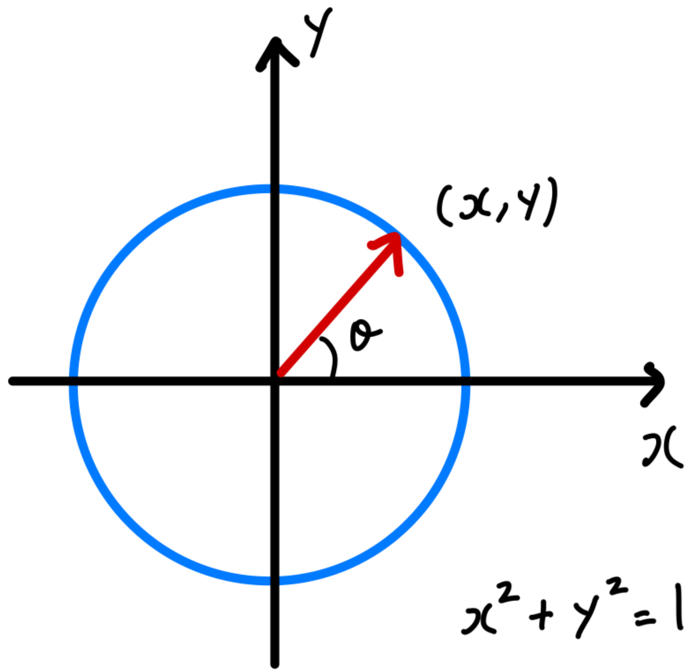
<2x2 Rotation Matrix>

It has a 1 DoF because it belongs to SO(2) group.

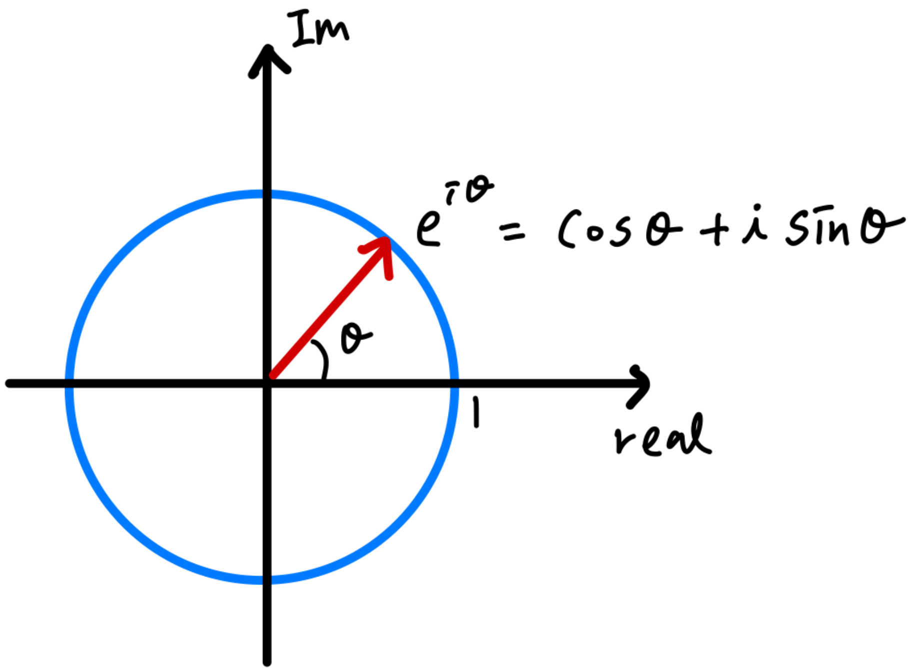
S: Special
O: Orthogonal
2: 2x2

Orientation: 2D

Complex Plane



<Cartesian Coordinate>



<Complex Plane>

Orientation: 2D

Euler Formula: Proof

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Differentiation

$f(x) = e^{-ix}(\cos x + i\sin x) \cdots (1)$ 라면,

$$\frac{d}{dx}f(x) = -ie^{-ix}(\cos x + i\sin x) + e^{-ix}(-\sin x + i\cos x) = e^{-ix}(-i\cos x + \sin x - \sin x + i\cos x) = 0$$

$\therefore f(x) = C$ (단, C는 상수)

(1)에 $x = 0$을 대입하면,

$f(0) = 1$

$\therefore C = 1$

$e^{-ix}(\cos x + i\sin x) = 1$

$e^{ix} = (\cos x + i\sin x)$

Q.E.D.

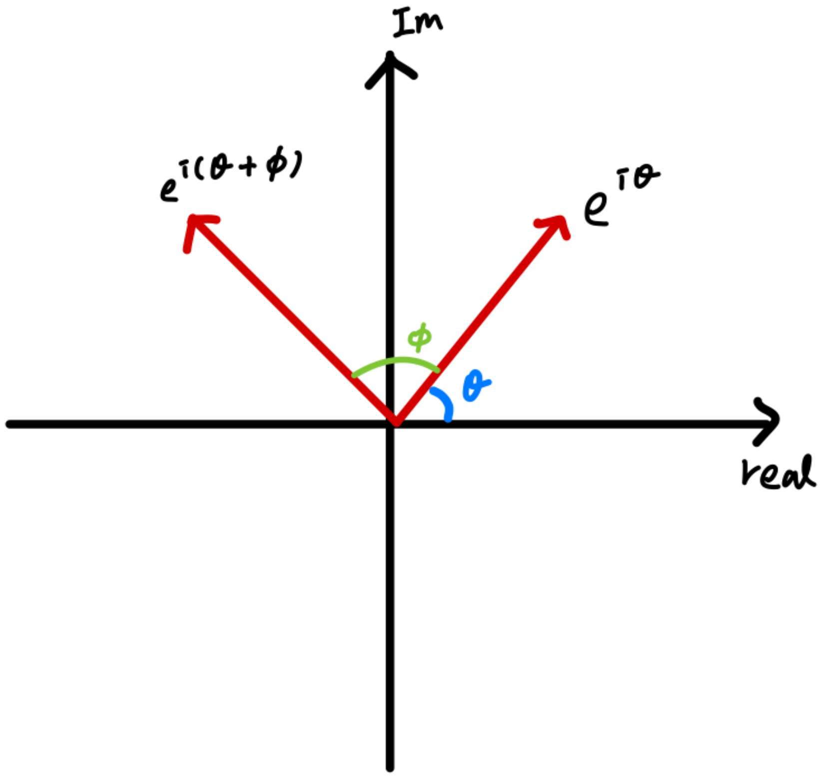
Taylor Expansion

$$\begin{aligned} e^{iz} &= 1 + iz + \frac{(iz)^2}{2!} + \frac{(iz)^3}{3!} + \frac{(iz)^4}{4!} + \frac{(iz)^5}{5!} + \frac{(iz)^6}{6!} + \frac{(iz)^7}{7!} + \frac{(iz)^8}{8!} + \cdots \\ &= 1 + iz - \frac{z^2}{2!} - \frac{iz^3}{3!} + \frac{z^4}{4!} + \frac{iz^5}{5!} - \frac{z^6}{6!} - \frac{iz^7}{7!} + \frac{z^8}{8!} + \cdots \\ &= \left(1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{z^6}{6!} + \frac{z^8}{8!} - \cdots\right) + i\left(z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + \cdots\right) \\ &= \cos z + i\sin z \end{aligned}$$

Orientation: 2D

Euler Formula

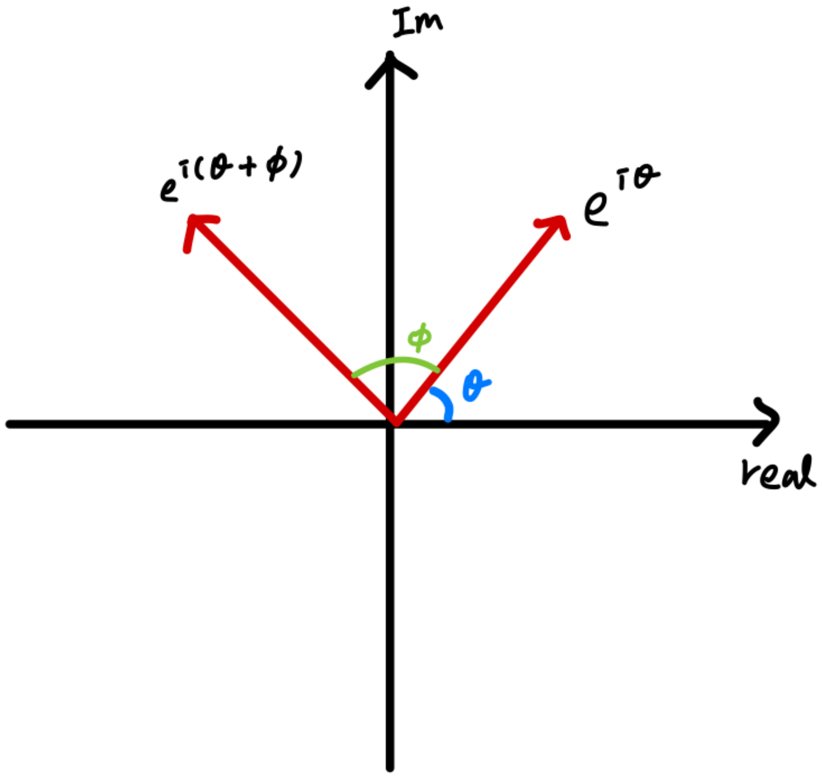
$$e^{i(\theta+\emptyset)} = e^{i\theta}e^{i\emptyset}$$



Orientation: 2D

Euler Formula

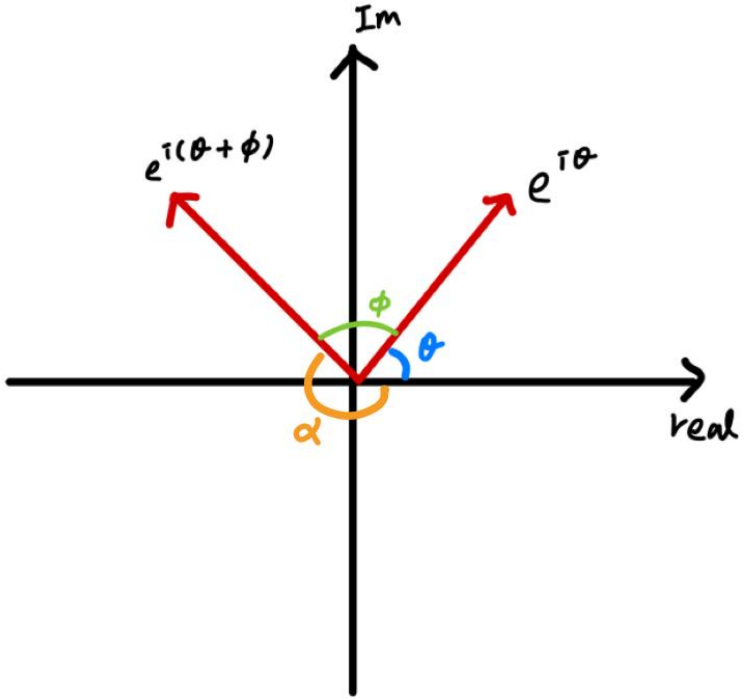
$$e^{i(\theta+\emptyset)} = e^{i\theta}e^{i\emptyset}$$



What is the problem?

Orientation: 2D

Complex Plane



$$e^{i(\theta+\emptyset)} = e^{i\alpha}$$

Advantage)

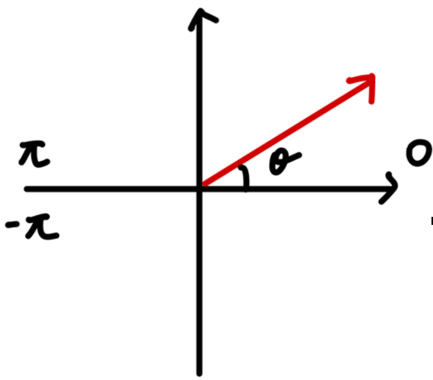
- There is no singularity.
- It can handle multiple rotations through the multiplication of complex numbers.

Dis-Advantage)

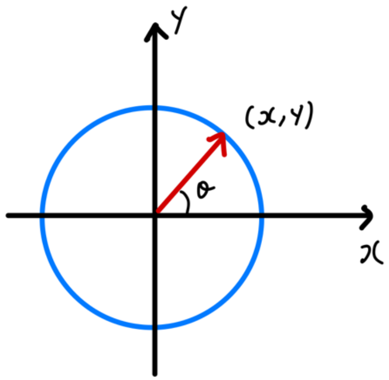
- The representation of Orientation is not unique.

Orientation: 2D

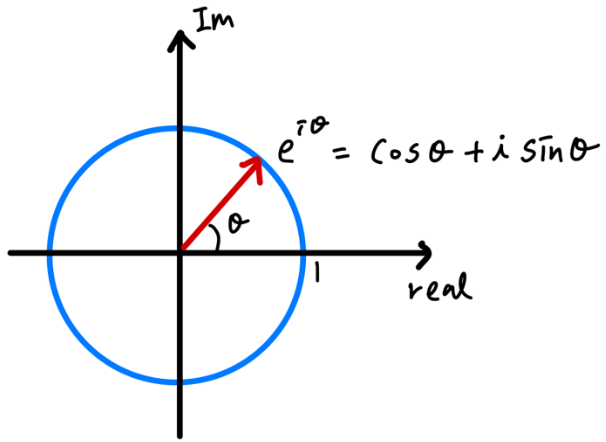
Summary



<Polar Coordinate>



<Cartesian Coordinate>



<Complex Plane>

$$e^{i(\theta+\emptyset)} = e^{i\theta}e^{i\emptyset}$$

<Euler Formula>

Next: 3D

Bonus Lecture

Motion capture dataset

DOI: 10.1111/cgf.14426

COMPUTER GRAPHICS *forum*
Volume 41 (2022), number 1 pp. 122–157

A Survey on Deep Learning for Skeleton-Based Human Animation

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[2]InterDigital, Inc, Cesson Sevigne, France

Abstract
Human character animation is often critical in entertainment content production, including video games, virtual reality or fiction films. To this end, deep neural networks drive most recent advances through deep learning (DL) and deep reinforcement learning (DRL). In this article, we propose a comprehensive survey on the state-of-the-art approaches based on either DL or DRL in skeleton-based human character animation. First, we introduce motion data representations, most common human motion datasets and how basic deep models can be enhanced to foster learning of spatial and temporal patterns in motion data. Second, we cover state-of-the-art approaches divided into three large families of applications in human animation pipelines: motion synthesis, character control and motion editing. Finally, we discuss the limitations of the current state-of-the-art methods based on DL and/or DRL in skeletal human character animation and possible directions of future research to alleviate current limitations and meet animators' needs.

Keywords: animation systems, human simulation, motion capture, physically based animation

CCS Concepts: • *General and reference → Surveys and overviews*; • *Applied computing → Media arts*; • *Computing methodologies → Motion processing; physical simulation; neural networks; supervised learning; unsupervised learning; reinforcement learning;*

Mourot, L., Hoyet, L., Le Clerc, F., Schnitzler, F. and Hellier, P. (2022),
A Survey on Deep Learning for Skeleton-Based Human Animation.
Computer Graphics Forum, 41: 122-157.
https://doi.org/10.1111/cgf.14426

Motion capture dataset

Which motion capture data should we choose among the many available?

Table 1: *Summary of the main datasets presented in Section 2.2.*

			Data			
Dataset	URL	Size	Joints	Representation	Miscalleneous	Availability
CMU [Uni03]	http://mocap.cs.cmu.edu	$3.9 * 10^6$ frames @ 120Hz →9.1 h	29	angular		Public
HDM05 [MRC*07]	http://resources.mpi-inf.mpg.de/HDM05	$3.6 * 10^5$ frames @ 120Hz →0.8 h	31	angular		Public
Human3.6M [IPOS14]	http://vision.imar.ro/human3.6m	$3.6 * 10^6$ frames @ 50Hz →20.0 h	32	angular	RGB+D	On request
Holden et al. [HSK16]	https://theorangeduck.com/page/deep-learning-framework-character-motion-synthesis-and-editing	$6.0 * 10^6$ frames @ 120Hz →13.9 h	21	positional		Public
NTU RGB+D [SLNW16]	https://rose1.ntu.edu.sg/dataset/actionRecognition	$4.0 * 10^6$ frames @ 30Hz →37.0 h	25	positional	RGB+D+IR	On request
NTU RGB+D 120 [LSP*20]	https://rose1.ntu.edu.sg/dataset/actionRecognition	$8.0 * 10^6$ frames @ 30Hz →74.1 h	25	positional	RGB+D+IR	On request
3DPW [vMHB*18]	https://virtualhumans.mpi-inf.mpg.de/3DPW	$5.1 * 10^4$ frames @ 30Hz →0.5 h	23	angular	RGB	Public
AMASS [MGT*19]	https://amass.is.tue.mpg.de	$1.8 * 10^7$ frames @ [60, 250]Hz →41.5 h	52	angular	Body mesh	Public
Mixamo [Ado]	https://www.mixamo.com/	$2.7 * 10^5$ frames @ 30Hz →2.5 h	52	angular	Body mesh	Public

Motion capture dataset

Which motion capture data should we choose among the many available?

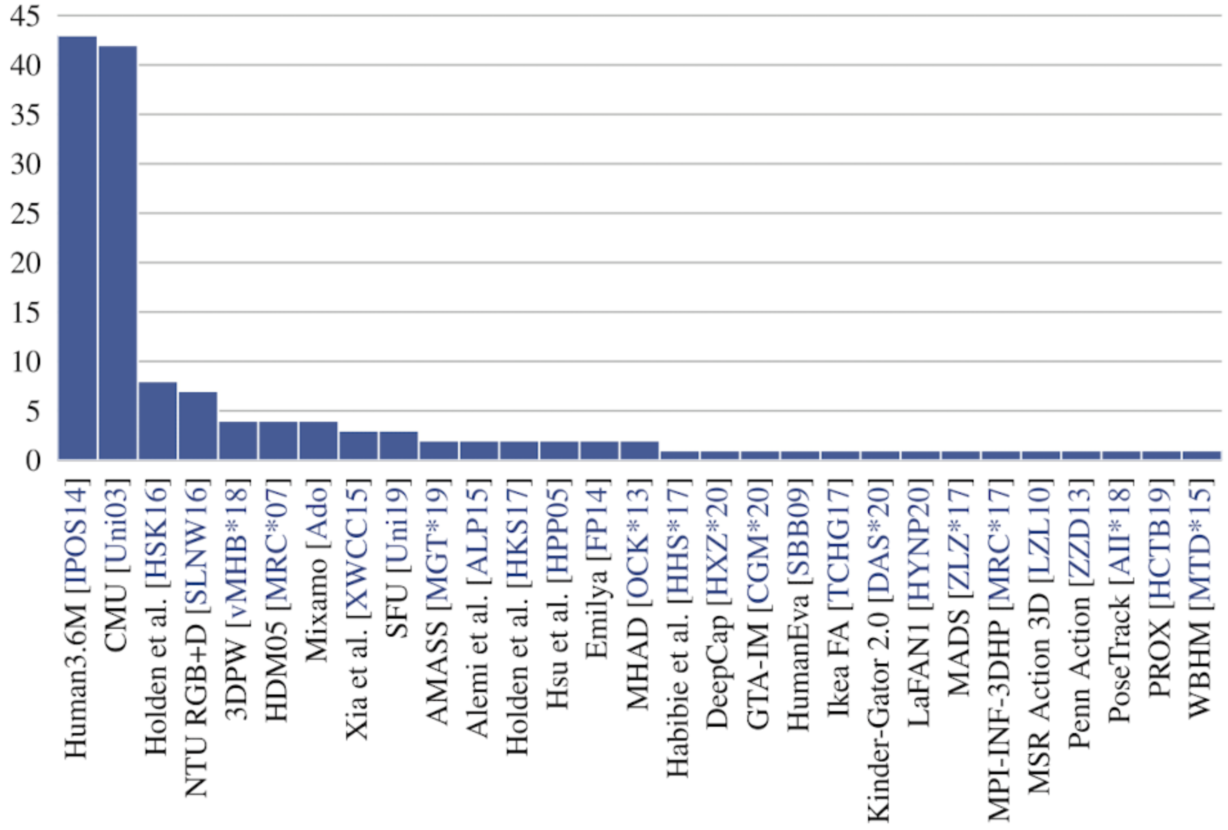
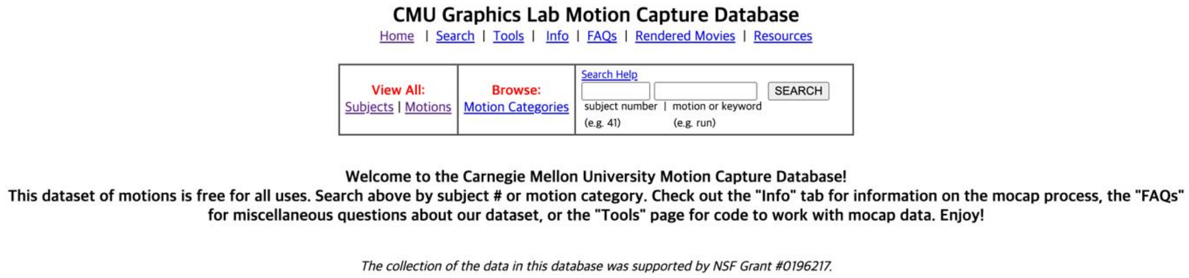


Figure 4: *Histogram of the number of papers using a given dataset among the works in skeleton-based deep human animation covered in this survey.*

Motion capture dataset

Which motion capture data should we choose among the many available?



Human3.6M

- http://vision.imar.ro/human3.6m/description.php
- Institue of Mathematics of the Romanian Academy
- 3.6M frames Large dataset
- 3D motion capture data, 2d image
- 4 views of camera

- public x

CMU Graphics Lab Motion Capture Database

- http://mocap.cs.cmu.edu/
- CMU Graphics Lab
- there are various motion tasks
- 144 subjects, #2605, 6categories
  23 subcategories
- 3D motion capture data
  (c3d, asf/amc, vsk.v, txt)

- public